OCR
Oxford Cambridge and RSA

.CG Maths.
Worked Solutions

Friday 19 May 2023 – Morning

GCSE (9–1) Mathematics

J560/04 Paper 4 (Higher Tier)

Time allowed: 1 hour 30 minutes

You must have:
- the Formulae Sheet for Higher Tier (inside this document)

You can use:
- a scientific or graphical calculator
- geometrical instruments
- tracing paper

H

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number | | | | | | Candidate number | | | | |

First name(s) ____________________

Last name ____________________

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided. You can use extra paper if you need to, but you must clearly show your candidate number, the centre number and the question numbers.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Use the π button on your calculator or take π to be 3.142 unless the question says something different.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **20** pages.

ADVICE
- Read each question carefully before you start your answer.

Please note that these worked solutions have neither been provided nor approved by OCR and may not necessarily constitute the only possible solutions. Please refer to the original mark schemes for full guidance.

Any writing in blue indicates what must be written in order to answer the questions and get the marks. The worked solutions have been designed to show the smallest amount of work which needs to be done to answer the question.

Anything written in green in a cloud doesn't have to be written in the exam.

Anything written in orange in a rectangle doesn't have to be written in the exam and is there to show what should be put into a calculator or measured using a ruler or protractor.

If you find any mistakes or have any requests or suggestions, please send an email to curtis@cgmaths.co.uk

1 Calculate.

$$\sqrt{5.2^2 - 4.8 \times {}^{-}6.3}$$

Give your answer correct to **3** significant figures.

Putting it in the calculator exactly as it is above. Then rounding the answer of 7.568355171 to 3 significant figures

........................7.57........................ **[2]**

2 The price of petrol decreases from £1.32 per litre to £1.02 per litre.

Calculate the percentage decrease in the price.

$\frac{1.02-1.32}{1.32} \times 100$

Subtracting the £1.32 from the £1.02 expresses the decrease in the price. Putting this over the original £1.32 expresses the decrease as a fraction. Multiplying this by 100 converts it to a percentage

The negative in the answer can be ignored as it is asking for percentage decrease

........................22.7........................ % **[3]**

3 **(a)** Eve, Jack and Ling share some money in the ratio 2 : 3 : 4.
Jack gets £720.

Work out how much Ling gets.

$\frac{720}{3} \times 4$ ← £720 is represented by 3 parts in the ratio. So dividing the £720 by 3 works out the value of 1 part of the ratio. Multiplying this by the 4 parts which represent what Ling gets works out how much Ling gets

(a) £ ....960........ **[2]**

(b) Amir, Beth and Casey share some money in the ratio 3 : 5 : *c*.
Casey's share is $\frac{2}{3}$ of the total.

Find the value of *c*.

$3+5$ ← This works out that there are 8 parts in the ratio which do not represent Casey

8×2 ← These 8 parts must be 1/3 of the total as 1 - 2/3 = 1/3. So multiplying it by 2 works out the number of parts representing 2/3 of the total, which is c

(b) *c* = ...16........ **[3]**

4 Alex invests £4500 at a rate of 7.5% per year simple interest.

(a) Find the value of the investment at the end of 4 years.

$\frac{7.5}{100} \times 4500$ ← This works out that the interest is £337.50 each year. Putting the 7.5% over 100 converts it into a fraction, which when multiplied by finds 7.5%

337.5×4 ← The interest is the same every year as it is simple interest. So multiplying the £337.50 by 4 works out that the interest over 4 years is £1350

$4500 + 1350$ ← Adding the total interest to the starting amount works out the value of the investment at the end of 4 years

(a) £ ......5850...... **[3]**

(b) At the end of *t* years, the value of the investment is over £13 500 for the first time.

Find the value of *t*.

$13500 - 4500$ ← Subtracting the starting amount from the £13500 works out that £9000 was gained in interest

$9000 \div 337.5$ ← Dividing the total interest by the £337.50 interest each year works out exactly how many years it would take and so works out t

The answer of $26.\dot{6}$ needs to be a whole number and 26 is not enough. So it should be rounded up to 27

(b) $t =$ ......27...... **[3]**

5 **(a)** Write 0.003 86 in standard form.

Multiplying by ten 3 times gives a decimal which is at least 1 and less than 10. It must be divided by ten 3 times to keep it equal. Multiplying by 10^{-3} does this

3.86×10^{-3} .... **[1]**

(b) The speed of sound is 3.43×10^{-1} km/s.
An object is travelling at the speed of sound.

Work out how far the object travels in one day.

$3.43 \times 10^{-1} \times 60 \times 60 \times 24$

3.43×10^{-1} km is travelled in 1 second. There are 60 seconds in a minute so multiplying this by 60 works out how far is travelled in a minute. There are 60 minutes in an hour so multiplying this by 60 works out how far is travelled in an hour. There are 24 hours in a day so multiplying this by 24 works out how far is travelled in one day

(b) ....29635.2.......... km **[2]**

(c) In a science fiction story, a spacecraft travelling faster than the speed of light is said to be travelling at 'warp *n*' where *n* is an integer.

Warp *n* is defined as $n^3 \times$ the speed of light.

In the story, a spacecraft needs to travel from Earth to Neptune in less than 2 minutes.

- The speed of light is 3.00×10^5 km/s.
- The distance from Earth to Neptune is 4.41×10^9 km.

Find the smallest possible warp *n* at which the spacecraft can travel.
You must show your working.

$$\frac{4.41 \times 10^9}{2 \times 60}$$

To work out the speed in km/s, the distance in km should be divided by the time in seconds. Multiplying the 2 minutes by 60 converts it into seconds as there are 60 seconds in a minute

$n^3 \times 3 \times 10^5 = 36750000$

Setting the speed in warp n equal to the speed needed in km/s

$n^3 = 122.5$

Dividing both sides by 3×10^5

Cube rooting both sides finds that n is 4.96... but the minimum it can be is 5 as it must be an integer

(c) $n =$ ....5.......... **[3]**

6 A six-sided numbered spinner is thrown 50 times.
The score for each throw is recorded.
Some of the results are shown in the table.

An 8 was thrown *f* times.
An unknown number on the spinner is represented by *n*.

Score	Frequency	
1 ×	12	= 12
3 ×	2	= 6
5 ×	9	= 45
6 ×	16	= 96
8 ×	*f*	= 56
n ×	4	= 4n
Total	50	

2. Multiplying the scores by the frequencies works out the total score for each category

The mean score of the 50 throws is 5.5 .

Find the value of *f* and the value of *n*.

$50-12-2-9-16-4$ ← 1. Subtracting all of the other frequencies from 50 works out that f = 7

$\frac{215+4n}{50}=5.5$ ← 3. Adding all of the total scores for each category gives 215 + 4n. Dividing this by the total frequency of 50 expresses the mean. This must be equal to the actual mean of 5.5

$215+4n=275$ ← 4. Multiplying both sides by 50 eliminates it as the denominator on the left

$4n=60$ ← 5. Subtracting 215 from both sides eliminates the 215 on the left and gets the n term on its own

6. Dividing both sides by 4 eliminates the 4 on the left and gets n on its own

f = ...7............

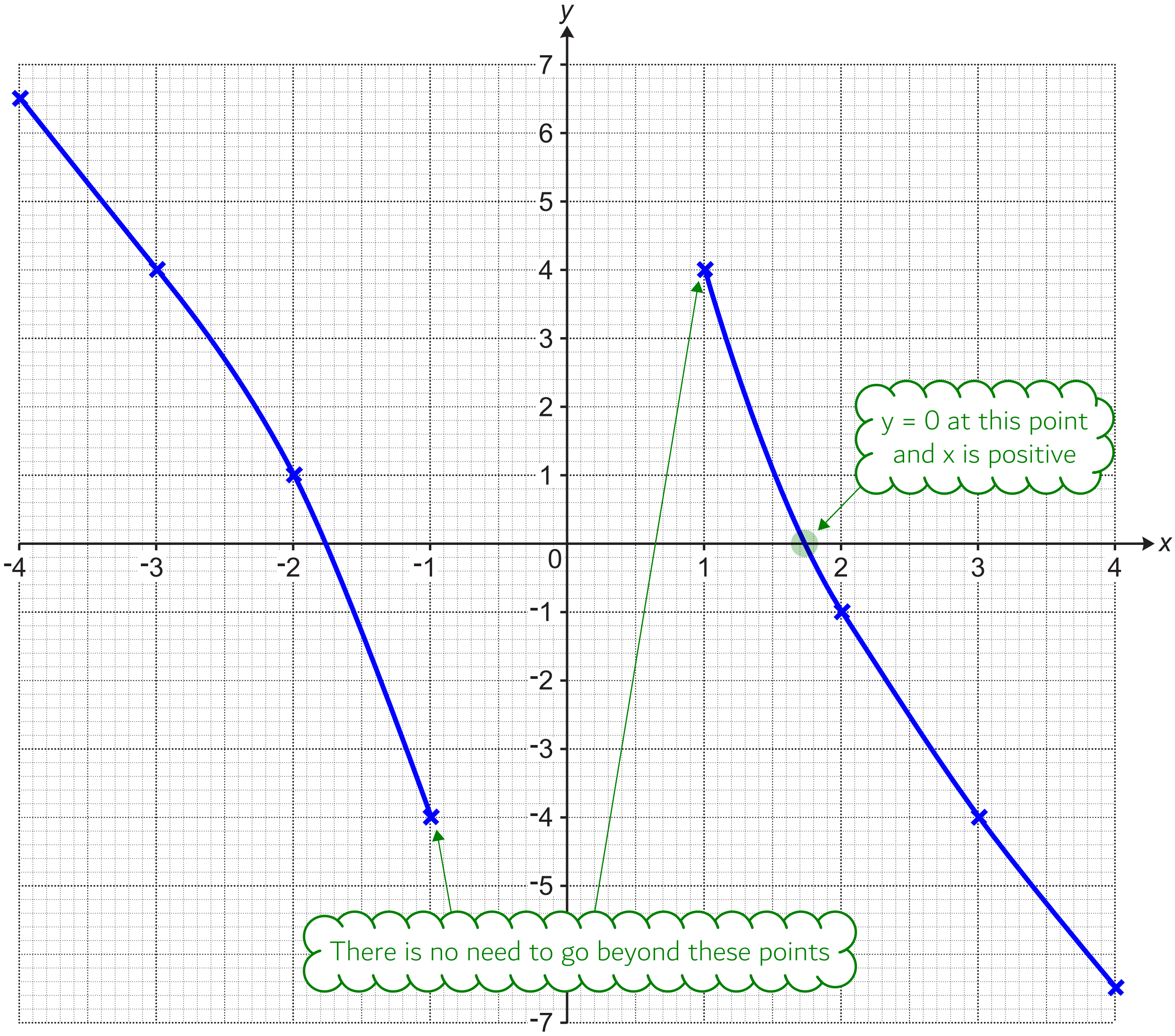
n = ...15............

[4]

7 Here is a table of values for $y = \frac{6}{x} - 2x$.

x	$^-4$	$^-3$	$^-2$	$^-1$	1	2	3	4
y	6.5	4	1	$^-4$	4	$^-1$	$^-4$	$^-6.5$

(a) Draw the graph of $y = \frac{6}{x} - 2x$ for $^-4 \leqslant x \leqslant 4$, $x \neq 0$.



[3]

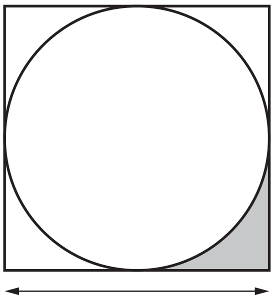
(b) Use your graph to find the positive solution of $\frac{6}{x} - 2x = 0$.

It is basically asking what x is when y = 0. x must be positive

Give your answer to **1** decimal place.

(b) $x =$ ....1.7.......... **[1]**

8 The diagram shows a circle inside a square of side 12 cm.



Work out the percentage of the square that is shaded.
You must show your working.

$12^2 = 144$ Area of square = length^2. So the area of the square is 144 cm^2

$12 \div 2$ The diameter of the circle is 12 cm. Dividing this by 2 works out that the radius is 6 cm

$\pi \times 6^2$ Area of circle = π x radius^2. So the area of the circle is 36π cm^2

$144 - 36\pi$ Subtracting the area of the circle from the area of the square works out the area of 4 lots of the shaded area

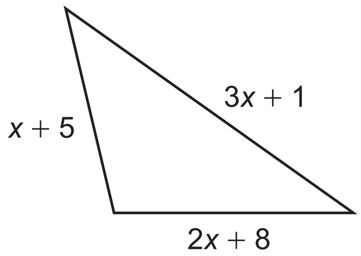
$30.9... \div 4$ Dividing the area of 4 lots of the shaded area by 4 works out the shaded area

$\frac{7.7...}{144} \times 100$ Expressing the shaded area as a fraction of the area of the square. Converting it into a percentage by multiplying by 100

Rounding the exact answer of 5.365045915

5.4 .... % **[6]**

9 The sides of this triangle are given in centimetres.
The perimeter of the triangle is 80 cm.



Not to scale

(a) Find the length of each side of the triangle.
You must show your working.

$x+5+3x+1+2x+8$ Expressing the perimeter by adding all of the sides

$6x+14=80$ Simplifying the expression by collecting like terms and setting equal to the actual perimeter of 80 cm

$6x=66$ Subtracting 14 from both sides

$x=11$ Dividing both sides by 6

$11+5$
$3\times 11+1$
$2\times 11+8$
Substituting the value of x back into the expressions of the side lengths finds their values

(a) ....16........ cm, ...34........ cm and ....30........ cm **[5]**

(b) Is the triangle above a right-angled triangle?
Use calculations to show how you decide.

$a^2+b^2=c^2$ Writing down Pythagoras' Theorem, where a and b are the two shorter sides and c is the longest side

$\sqrt{16^2+30^2}=34$ Square rooting both sides makes c the subject. Then substituting in 16 for a and 30 for b works out that c would be 34 if it was right-angled

.......Yes....... because ....Pythagoras' Theorem works....

The longest side was calculated to be 34, which is the same as the actual longest side

.... **[3]**

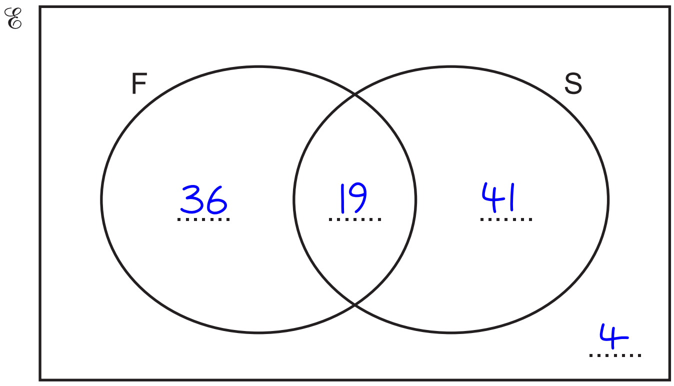
.CG Maths.

10 100 people were asked whether they had visited France (F) or Spain (S).

- 55 had visited France
- 60 had visited Spain
- 4 had not visited either country.

55 + 60 + 4 = 119. This is 19 more than the 100 people so 19 must go in the middle. 55 - 19 = 36 who only visited France. 60 - 19 = 41 who only visited Spain

(a) Complete the Venn diagram.



[3]

(b) One of these 100 people is chosen at random.

(i) Write down the probability that this person had visited exactly **one** of the countries.

Adding the 36 who only visited France and the 41 who only visited Spain works out that 77 out of the 100 people visited exactly one of the countries

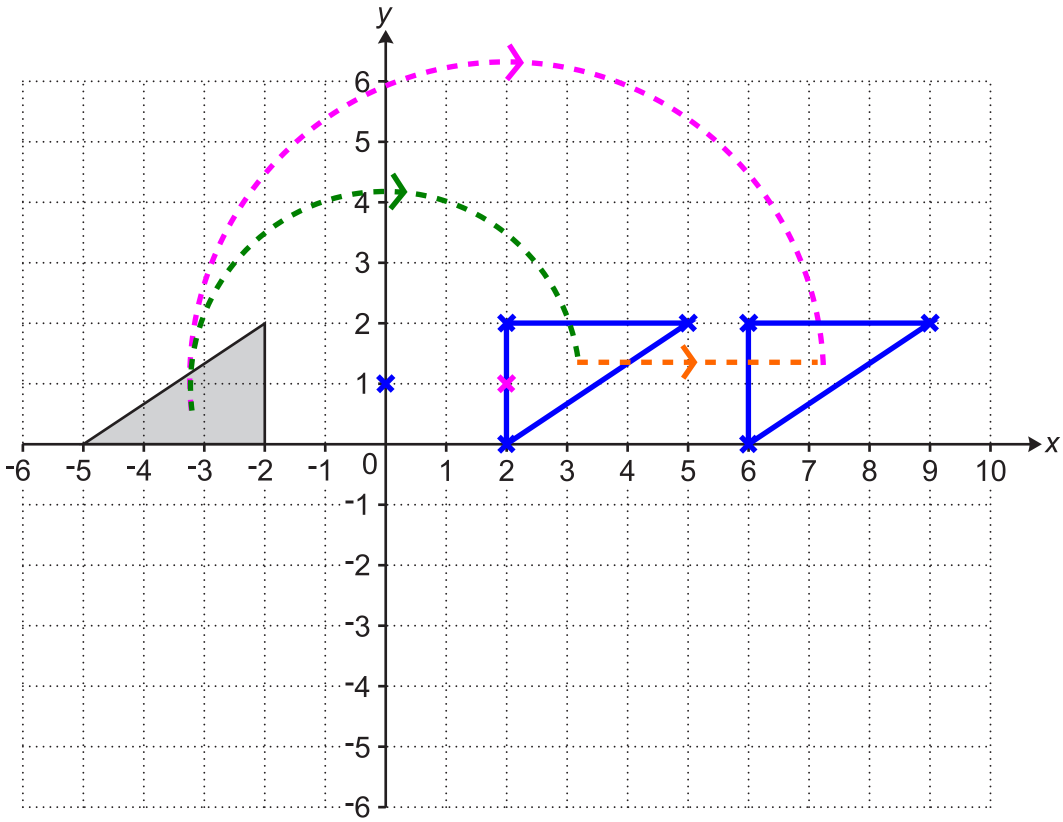
(b)(i) $\frac{77}{100}$ **[2]**

(ii) Write down the probability that this person had visited France given that they had also visited Spain.

19 + 41 = 60 people who had visited Spain. Out of these, 19 had visited France

(ii) $\frac{19}{60}$ **[2]**

11 You may use this coordinate grid to help you answer the following question.



Describe fully the **single** transformation that is equivalent to:

- a rotation of 180° with centre (0, 1), followed by
- a translation of $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$.

Rotation of 180° with centre (2 , 1) ....

.... **[3]**

Using tracing paper, rotate the original triangle 180° centre (0 , 1) shown by the green dashed line. Then translate it 4 to the right shown by the orange dashed line. The pink dashed line shows the single transformation which is equivalent to this

12 Here are two functions.

Function A: Input $\longrightarrow$ $\boxed{+\,5}$ $\longrightarrow$ $\boxed{\times\,3}$ $\longrightarrow$ Output

Function B: Input $\longrightarrow$ $\boxed{\times\,2}$ $\longrightarrow$ $\boxed{-\,1}$ $\longrightarrow$ Output

(a) Write an algebraic expression for the output of function A when the input is x.

Brackets are necessary so that the +5 is done first. Otherwise the order of operations (BIDMAS) would mean that the multiplication is done first

(a) $(x+5)\times 3$ **[1]**

(b) Here is a composite function C.

Function C: Input $\longrightarrow$ $\boxed{\text{Function A}}$ $\longrightarrow$ $\boxed{\text{Function B}}$ $\longrightarrow$ Output

The input to function C is x.
The output from function C is $2x + 1$.

Find the value of x.
You must show your working.

$3x+15$ ← Expanding the bracket for the expression of the output of function A when the input is x which was written in part (a)

$6x+30$ ← Multiplying by 2 as this is the next step of function B

$6x+29=2x+1$ ← Subtracting 1 as this is the next step of function B. Setting this output equal to the output of 2x + 1

$4x+29=1$ ← Subtracting 2x from both sides to get all of the x on the same side

$4x=-28$ ← Subtracting 29 from both sides to get the x term on its own

Dividing both sides by 4 gets x on its own

(b) $x =$ -7 **[5]**

13 Here are two pieces of work.

Each shows a question and the **first line** of an incorrect solution.

For each part, describe the error made in the first line of the solution.
You do **not** need to complete the solution.

(a)

Question:
Simplify. $\frac{2}{x-1}+\frac{3}{x+4}$

Solution:
$$\frac{2}{x-1}+\frac{3}{x+4}=\frac{2(x-1)+3(x+4)}{(x-1)(x+4)}$$

The numerator should be multiplied by the same as what the denominator is multiplied by when making the denominators the same. Then the numerators can be added and the denominator stays the same. The denominator of the first fraction is multiplied by x + 4 so the numerator should have also been multiplied by this

The numerator should be 2(x + 4) + 3(x - 1)

.... **[1]**

(b)

Question:
Solve. $x^2 + 7x + 5 = 0$

Solution:
$$x = -\frac{7 \pm \sqrt{7^2 - 4\times1\times5}}{2\times1}$$

This question was discounted and all candidates were awarded 1 mark

There should not be a negative outside of the fraction

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ... The negative should be applied to the first 7 on the numerator ... **[1]**

14 A college offers 41 different subjects including 9 different languages.
Students are asked to choose one subject from Option A, one subject from Option B and one subject from Option C.

Each of the 41 different subjects appears only once, either in Option A, or in Option B or in Option C.

Option A : 14 subjects including 2 languages
Option B : 12 subjects including 3 languages
Option C : 15 subjects including 4 languages

Work out the proportion of all the possible subject combinations that include **at least one** language.
You must show your working.

$14 \times 12 \times 15 = 2520$

Using the product rule for counting works out that there are 2520 possible subject combinations

$(14-2)(12-3)(15-4)$

Using the product rule for counting works out that there are 1188 possible subject combinations without any languages. Subtracting the number of languages from the number of subjects leaves the number of subjects without languages for each option.

$2520 - 1188$

Subtracting the 1188 possible subject combinations without any languages from the 2520 possible subject combinations leaves 1332 possible combinations with at least one language

1332 out of the possible 2520 combinations include at least one language

$\frac{1332}{2520}$ **[5]**

15 y is inversely proportional to the square root of x.
$y = 7$ when $x = 144$.

Find the value of y when $x = 16$.

$y = \frac{k}{\sqrt{x}}$

Writing the proportion as an equation by multiplying the right side by k, which represents an unknown value

$k = y\sqrt{x}$

Rearranging to find k by multiplying both sides by $\sqrt{x}$

$= 7\sqrt{144}$

Substituting in the values of x and y given finds that k is 84

$y = \frac{84}{\sqrt{16}}$

Substituting the value of k and the next value of x into the equation to find y

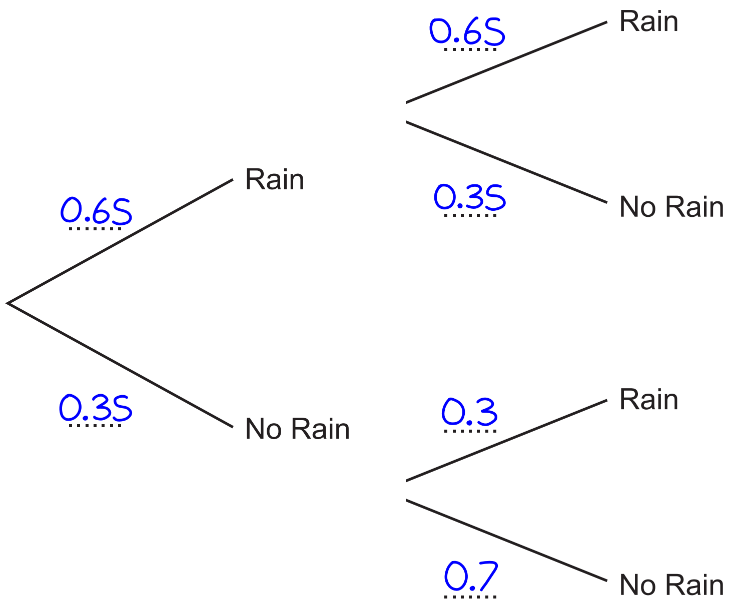
$y =$ 21 **[3]**

16 If it rains on a given day the probability that it will rain the next day is 0.65.
If it does **not** rain on a given day the probability that it will rain the next day is 0.3.
It rained on Monday.

(a) Complete the tree diagram.

The probabilities on each branch must add up to 1 as it is certain to either rain or not rain. 1 - 0.65 = 0.35 and 1 - 0.3 = 0.7

Tuesday **Wednesday**



[2]

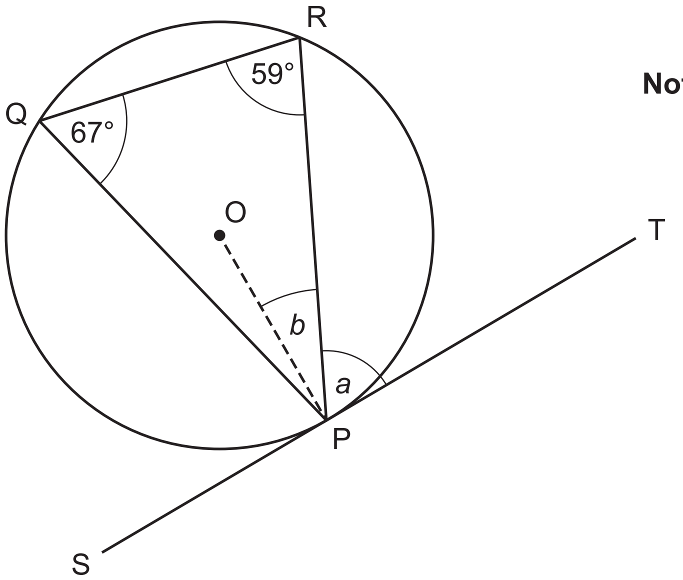
(b) Find the probability that it rains on Wednesday.

$0.65 \times 0.65 + 0.35 \times 0.3$

Rain on Tuesday AND rain on Wednesday OR no rain on Tuesday AND rain on Wednesday. AND means to multiply the probabilities, OR means to add the probabilities

(b) ....0.5275........ **[3]**

17 **(a)** P, Q and R are points on the circumference of a circle, centre O.



Not to scale

Angle PRQ = 59° and angle PQR = 67°.
Line SPT is a tangent to the circle.

(i) Work out angle *a*.
Give a reason for your answer.

Angle *a* = ...67........° because of the alternate segment theorem

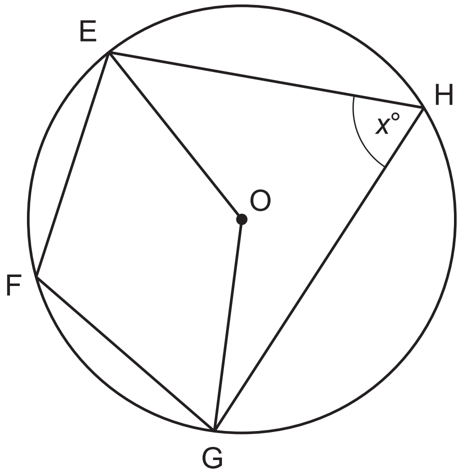
Alternate segment theorem: the angle between a tangent and a chord is equal to the interior opposite angle **[2]**

(ii) Work out angle *b*.
Give a reason for your answer.

Angle *b* = ...23........° because the angle between a tangent and a radius is 90°

Angle OPT = 90° and subtracting angle a leaves angle b. 90 - 67 = 23 **[2]**

(b) E, F, G and H are points on the circumference of a circle, centre O.
Acute angle EHG $= x°$.



Not to scale

(i) Complete the following, giving the values of the angles in terms of x.

Obtuse angle EOG = 2x ° because the angle at the centre is double the angle at the circumference

Therefore, reflex angle EOG = 360 - 2x °

As angles around a point add up to 360°

Therefore, angle EFG = 180 - x °

As the angle at the circumference is half the angle at the centre

[3]

(ii) Write down what your working in part **(b)(i)** has proved.

Opposite angles in a cyclic quadrilateral add up to 180°

As angles EHG and EFG are opposite angles in the cyclic quadrilateral EFGH and x + 180 - x = 180

[1]

18 **(a)** Show that the equation $x^3 + x^2 - 5 = 0$ has a solution between $x = 1$ and $x = 2$. **[3]**

$1^3+1^2-5=-3$

Substituting 1 for x in the left side of the equation gives -3

$2^3+2^2-5=7$

Substituting 2 for x in the left side of the equation gives 7

$-3<0<7$

0 is between -3 and 7. As it is a continuous function it must be equal to 0 between x = 1 and x = 2

(b) Find this solution correct to **1** decimal place.
You must show calculations to support your answer.

Using table mode on the calculator: f(x) = $x^3 + x^2 - 5$. Start: 1. End: 2. Step: 0.1. This substitutes in all the values between x = 1 and x = 2 to 1 decimal place into the left side of the equation

$1.4^3+1.4^2-5=-0.2...$
$1.5^3+1.5^2-5=0.6...$

There is a change in sign between x = 1.4 and x = 1.5 so the solution must be between these two values

Using table mode on the calculator: f(x) = $x^3 + x^2 - 5$. Start: 1.4. End: 1.5. Step: 0.01. This substitutes in all the values between x = 1.4 and x = 1.5 to 2 decimal places into the left side of the equation

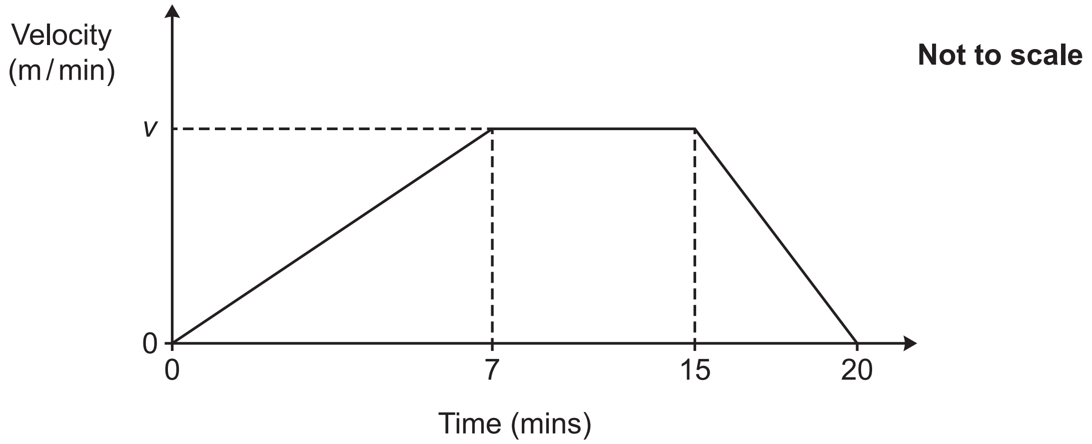
$1.43^3+1.43^2-5=-0.03...$
$1.44^3+1.44^2-5=0.05...$

There is a change in sign between x = 1.43 and x = 1.44 so the solution must be between these two values

A number between 1.43 and 1.44 must round to 1.4 to 1 decimal place

(b) $x =$ 1.4 **[4]**

19 The graph shows the velocity of a particle over the first 20 minutes of its motion.



Between 7 minutes and 15 minutes the velocity of the particle is *v* metres per minute.
The average velocity of the particle over the 20 minutes is 11.55 metres per minute.

Find the value of *v*.
You must show your working.

$\frac{1}{2}\times 7\times V+8\times V+\frac{1}{2}\times 5\times V$

The area under the line is the distance on a velocity-time graph. So adding all the areas expresses the distance travelled

Area of the second triangle. Area of triangle = 1/2 x base x height

Area of the rectangle. Area of rectangle = base x height

Area of the first triangle. Area of triangle = 1/2 x base x height

$3.5V+8V+2.5V$

Simplifying the expression of the total area

$\frac{14V}{20}=11.55$

Collecting like terms to fully simplify the expression of the total area. This is the distance in metres. Metres per minute means to divide the distance in metres by the time taken in minutes. So dividing this total distance by the 20 minutes expresses the velocity in metres per minute, which must be equal to the given velocity of 11.55 metres per minute

$14V=231$

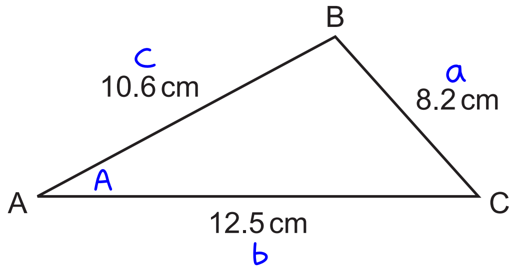
Multiplying both sides by 20 to eliminate the denominator on the left

Dividing both sides by 14 gets v on its own

v = ...16.5........ **[5]**

TURN OVER FOR QUESTION 20

20 The diagram shows triangle ABC.



Not to scale

Labelling the triangle. Side a is opposite angle A. The other two sides can be b and c

AB = 10.6 cm, BC = 8.2 cm and AC = 12.5 cm.

(a) Show that angle BAC = 40.5°, correct to **1** decimal place. **[3]**

$8.2^2 = 12.5^2 + 10.6^2 - 2 \times 12.5 \times 10.6 \times \cos A$

Substituting all the values into the cosine rule. $a^2 = b^2 + c^2 - 2bc\cos A$

$A = \cos^{-1}\left(\frac{8.2^2 - 12.5^2 - 10.6^2}{-2 \times 12.5 \times 10.6}\right)$

Rearranged to find A by subtracting the 12.5^2 and 10.6^2 from both sides, dividing both sides by the -2 x 12.5 x 10.6 then doing the inverse cos of both sides

$= 40.54\ldots$

The answer rounds to 40.5 to 1 decimal place. Storing the exact value on the calculator as it is needed for (b)

(b) Work out the area of triangle ABC.

$\frac{1}{2} \times 12.5 \times 10.6 \times \sin 40.54\ldots$

Area of triangle = 1/2 absinC, where a and b are two sides and C is the angle between them. Using the exact stored value as the angle

(b) ....43.1.......... cm^2 **[2]**

END OF QUESTION PAPER